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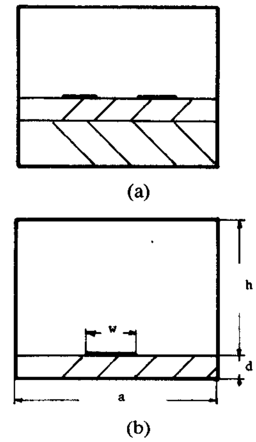

Fig. 1. (a) Cross-section view of planar MIC structure analyzable by the computer program of [4]. (b) Shielded microstrip line.

An Accurate Bivariate Formulation for Computer-Aided Design of Circuits Including Microstrip

Y. L. CHUA, J. B. DAVIES, AND D. MIRSHEKAR-SYAHKAL

Abstract —**An accurate and fast bivariate interpolation technique is used to compute the microstrip parameters at an arbitrary frequency and of any strip width. This technique allows computation of the effective dielectric constant, characteristic impedance, dielectric loss, and the conductor loss of microstrip in a time appropriate for computer-aided design application. By combining interpolation techniques with a highly accurate theory, computing is more accurate or faster than earlier theories, which achieve speed of computation by *a priori* approximations.**

I. Introduction

Various theories exist for microstrip and related planar lines which are 'exact-in-the-limit.' Properly implemented, these result in a computer program where for any analysis the designer can choose between an approximate, cheap result and an accurate, expensive result, the cost being measured in computer time and possibly computer storage. Similar to the approximate, cheap result, is the use of an *a priori* approximate theory, such as the many quasi-TEM theories, and approximate frequency-dependent theories [1], [2]. However, for interactive computer-aided design (CAD), and on other occasions, the time or cost of the accurate results may not be acceptable, and the designer has to make an awkward compromise.

The purpose of this paper is to show how the accurate results of a microstrip analysis program [3], [4] can be used to provide the data base for a subsequent program which in turn can give accurate and fast results over some specified range of parameters, such as frequency and strip width. It involves essentially bivariate interpolation over specified ranges, and as such, the method can be applied to many two-parameter problems. In this paper, the technique is illustrated with the accurate evaluation of four microstrip characteristics (phase velocity, characteristic impedance, attenuation due to conductor losses, and dielectric losses). Based on a published computer program [4], the computing times are reduced by a factor of a hundred or more in calculating these four characteristics over a continuous range of frequencies and of strip width.

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II. Theory

The theory of this paper is applied and illustrated with just the one basic computer program, one for the accurate analysis of microstrip, but its application to similar programs is implicit.

Reference [4] describes a program that considers the cross sections of Fig. 1(a), and for either single or coupled microstrip calculates the effective dielectric constant ϵ_{eff} (or phase velocity). Then, if required, it calculates the characteristic impedance Z_c, attenuation due to imperfect conductor α_c, and attenuation due to imperfect dielectric α_d.

Though the program is efficient in its class, it is still slow for CAD purposes. If results of the program ϵ_{eff}, Z_c, α_c, and α_d are obtained over the range of specified strip widths w and frequencies f, sets of this data can be considered as bivariate functions of w and f. We assume that the designer has control of, or needs results of, continuous parameters w and f, whereas the dielectric thickness and permittivity are of discrete values dictated by the substrate manufacturer. Other parameters affecting the results are the height and width of the conducting enclosure. These can be fixed at certain acceptable dimensions, possibly large enough to have negligible influence on ϵ_{eff}, etc.

Having generated the data sets, the objective is to find a suitable bivariate interpolation scheme which can accurately and efficiently give the values of ϵ_{eff}, Z_c, α_c, and α_d at any (w, f) values—not just at the data set points. Since accuracy and high efficiency are prerequisites for the interpolating method, the 'spline' technique using a 'tensor product' algorithm has been found to fulfill the requirements [5]. There are other methods for interpolation in one dimension, but their effectiveness for two-dimensional problems is subject to dispute [6].

Spline interpolation by means of the basis-spline function is a relatively new technique. Evolved through research on piecewise polynomial interpolation, it has gained importance in numerical analysis. It is widely used in computer graphic software, where extra smoothness, fast system response, and good interpolating accuracy are needed [7]. To interpolate with spline functions, details can be found in [5], [7]. However, a brief account seems appropriate for its use for the microstrip line.

We consider the one-dimensional spline technique first, the

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bivariate spline being obtained as a tensor product from the two one-dimensional spaces.

Consider a set of data points

$$\{g(t_i): i=1,2,\cdots,n\} \tag{1}$$

over the interval $[a, b]$ of the variable t, and let $\{x_1, x_2, \cdots, x_{n+1}\}$ denote a set of 'break-points' where

$$x_1 < a = t_1 < x_2 < t_2, \cdots, t_{n-1} < x_n = b < x_{n+1}. \tag{2}$$

Suppose that the interpolant function $f(y)$ is defined by a piecewise continuous set of polynomials of kth degree such that

$$f(y) = P_i(y) \quad \text{for} \quad x_i \leqslant y \leqslant x_{i+1}. \tag{3}$$

The above technique is called 'piecewise polynomial interpolation' and offers several advantages over conventional techniques. The main feature of (3) is that, for a function $f(y)$ with r continuous derivatives $(k > r+1)$, the interpolation error is reduced by increasing k and/or decreasing the interval ranges $[x_i, x_{i+1}]$. Although an increase in k theoretically improves the error, choosing a large k value is not always advisable, due to round-off errors; the high condition number connected with high degree polynomials can give rise to drastic errors [8]. In piecewise polynomial interpolation, high degrees can be avoided by use of suitably small intervals, and round-off error can be kept limited.

The break-points $\{x_i\}$ are chosen according to the problem requirements, and can even be taken as coincident with the points $\{t_i\}$. However, it should be noticed that often, interpolating at points *between* rather than *at* the break-points enhances the stability of the solution and makes the interpolation function insensitive to the end conditions [5].

To obtain the interpolation coefficients, one must decide on boundary conditions to be applied at the breakpoints, and different polynomials result from different choices. The orthodox kth-order spline interpolation is a piecewise polynomial that has $(k-2)$ continuous derivatives at its breakpoints. This continuity of derivatives has the advantage of giving smooth characteristics without being reduced to a single polynomial.

The maximum smoothness given by the spline interpolation is not always required, and in order to overcome the shortcomings, a new representation has been developed [5]. With this approach, the function $f(y)$ is given by

$$f(y) = \sum_i \alpha_i B_{i,k,r}(y) \tag{4}$$

where $B(y)$ is called the ith basis-spline function of order k for the nondecreasing knot sequence $\{s_i\}$, and α_i is the basis-spline coefficient. The knot sequence is obtained through consideration of the number of continuity conditions to be applied at the breakpoints.

As mentioned earlier, the one-dimensional spline interpolation of (4) is not adequate for our problem, but the theory can be extended to our two-dimensional case. An efficient way of achieving a surface spline interpolation is through the tensor product of the basis functions $B_{i,k,r}(y)$ and $B_{j,l,s}(z)$ in the y and z coordinates. The theory is now applied to the microstrip problem. The procedure is explained through the following numerical example.

III. Numerical Results

For the microstrip shown in Fig. 1(b), the parameters ϵ_{eff}, Z_c, α_c, and α_d have been computed by the program in [4], over the range of frequencies 1 to 9 GHz, at intervals of 1 GHz. The strip width w has been considered at values of 0.1, 0.2, $\cdots$, 1.1 mm. The data set for ϵ_{eff} that results is plotted in Fig. 2. Similar data sets for Z_c, α_c, and α_d are computed but not illustrated.

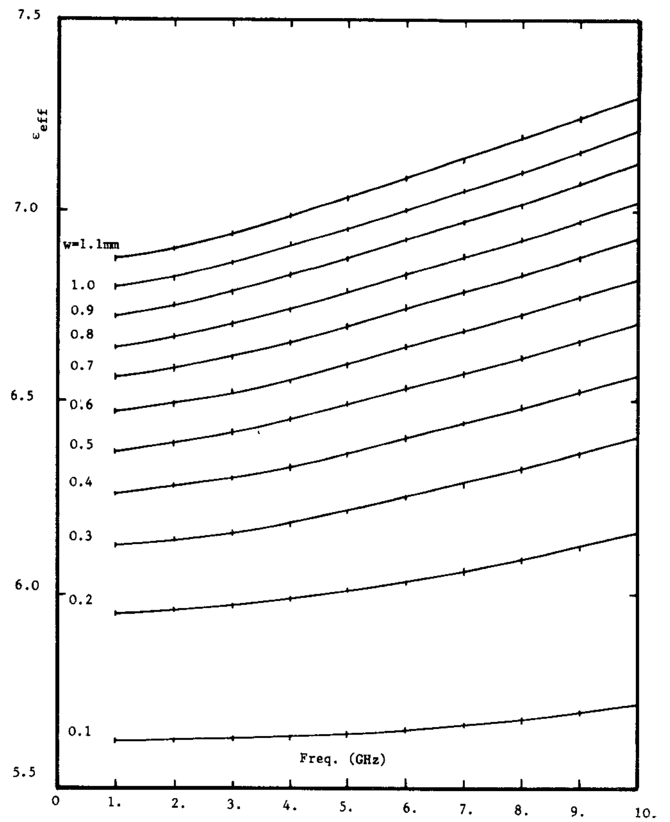

Fig. 2. Dependence of effective dielectric constant $(c/v_p)^2$ on frequency and microstrip width. Data is for substrate dielectric constant of 9.7, $d = 0.635$ mm, $h = 19.365$ mm, and $a/w = 20$ (in notation of Fig. 1(b)).

TABLE I
Effective Dielectric Constant Computed Directly, and Via Interpolation, at "Random" Values of Frequency and Strip Width*

Freq. (GHz)	w (mm)	ϵ_{eff} direct	ϵ_{eff} interpolated
6.88800	0.591600	6.66499	6.66473
4.55600	0.296600	6.19397	6.19381
8.54200	0.696800	6.84832	6.84855
8.63700	0.649300	6.80318	6.80358
6.34200	0.537600	6.58220	6.58264
3.48500	0.171900	5.90373	5.89780
8.13500	0.590400	6.71788	6.71784
6.29100	0.636800	6.68612	6.68653
3.81100	0.684000	6.62883	6.62881
3.89500	0.224900	6.04223	6.04656

*The geometry is that of Fig. 1(b), with the parameters of Fig. 2.

These four generated data sets form the data-base for the two-dimensional (w and f) interpolation computer program, based on the theory of Section II. The basis of the program is described in [5], and our problem is arranged in two parts. In the first (which is executed just once for a given range of parameters) the spline coefficients, α_i of (4), are computed from the above described data set. The second part then uses the computed coefficients as a regular data base and generates very quickly the microstrip parameters ϵ_{eff}, Z_c, α_c, and α_d for any provided w and f.

The two-dimensional interpolation for this problem has been set to include cubic spline functions of f, and parabolic spline functions of w. For the w-dependence the interpolation is carried out *between* knots while for the f-dependence it has been arranged *at* knots. As mentioned in the preceding section, these arrangements raise the accuracy and stability of the algorithm. The orders of the spline functions were experimentally determined for the desired stability, and to give a reasonable compromise between CPU time and accuracy.

Table I displays some of the exact data set for ϵ_{eff} alongside values computed by the bivariate interpolation program. The frequencies and widths were successive 'pseudo-random' numbers generated over the valid ranges 1 to 9 GHz and 0.1 to 1.1 mm, and rounded to 4 decimal figures before use as data for the test. The differences in this Table indicate that the interpolation error is indeed very small, with an rms error of less than 0.04 percent. Through experiments with the program, it was found that the CPU time for computing a set of microstrip parameters is at least 100 times less that of the original program. The final CPU time is then of order tens or hundreds of milliseconds, depending on the range of parameters, the computer, etc.

IV. Conclusions

The lack of accuracy in earlier approximate solutions for the microstrip line gives the motive to find a new, accurate, and fast technique for computation in times appropriate for CAD purposes. A bivariate interpolation has been used to compute all the microstrip parameters with high accuracy and efficiency. The data base is provided in a 'once-for-all' computer run with a very accurate program and analysis. In subsequent computation, perhaps as part of an interactive or automated design procedure, the bivariate interpolation makes use of the basis-spline functions and the tensor product.

Following this same technique, described for microstrip, other planar lines can be similarly programmed. Interpolation by basis-spline functions becomes distinctly superior for lines (or circuits) whose characteristics are not easily approximated on physical, *a priori* grounds.

Although this bivariate interpolation is adequate for many circumstances, circuits with more than two 'continuous variables' (such as strip width, strip separation, and frequency of a microstrip directional coupler) need a more general multivariate algorithm. To the authors' knowledge, no such efficient algorithm has yet been established.

Postscript

Referees have suggested possible publication of the associated computer programs. The two programs have no documentation suitable for publication, but copies of the extant programs (in Fortran IV) may be obtained from J. B. Davies.

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A Technique for Measuring the Effective Dielectric Constant of a Microstrip Line

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Abstract —**A method is discussed by which the effective dielectric constant of a transmission line of complex cross section is determined experimentally.**

In an effort to determine the insertion loss represented by a microwave filter, it becomes necessary to determine the effective dielectric constant and the characteristic impedance of an unusual type of microstripline. The microstripline, as seen in Fig. 1, consists of a metal strip mounted on a dielectric slab which, in turn, is suspended over a ground plane.

Since this type of microstrip structure does not appear to be amenable to conventional microstrip formulas, it seems that a combination of theoretical and experimental techniques would be necessary in order to predict the line's parameters. Since the purpose of this letter is to emphasize the experimental techniques, only a brief summary will be given on the theoretical aspects. To theoretically determine the effective dielectric constant and characteristic impedance a variational technique developed by E. Yamashita and R. Mittra [1] is used, along with standard transmission-line-type formulas. To be more specific, once the distributed capacitance is determined for the inhomogeneously layered line ($\epsilon_s \neq 1$) (see Fig. 1), and the homogeneous line ($\epsilon_s = 1$), the ratio of these quantities gives the effective dielectric constant. With the effective dielectric constant known, the characteristic impedance can then be approximated by using conventional transmission-line formulas along with the homogeneous ($\epsilon_s = 1$) capacitance, which is determined by the variational formulation previously mentioned.

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